



SS – 345

V Semester B.A./B.Sc. Examination, Nov./Dec. 2018  
(Semester Scheme)  
(Fresh + Repeaters) (CBCS) (2016-17 and Onwards)  
Mathematics  
MATHEMATICS – V

Time : 3 Hours

Max. Marks : 70

*Instruction : Answer all questions.*

PART – A

1. Answer any five questions : (5×2=10)

- In a ring  $(R, +, \cdot)$ , show that  $a \cdot (-b) = (-a) \cdot b = -(a \cdot b) \forall a, b, \in R$ .
- Define subring of a ring and give an example.
- Show that the set of even integers is an ideal of the ring of integers.
- Find the unit normal vector to the surface  $(x - 1)^2 + y^2 + (z + 2)^2 = 9$  at  $(3, 1, -4)$ .
- If  $\phi = 2x^3y^2z^4$ , then find  $\nabla\phi$ .
- Write the Newton's divided difference interpolation formula.
- Evaluate  $\Delta^{10} (1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)$ .
- State the Trapezoidal rule for the integral  $\int_a^b f(x)dx$ .

PART – B

Answer two full questions. (2×10=20)

- Prove that the intersection of any two subrings is a subring. Give an example to show that the union of two subrings of a ring need not be a subring.
  - Prove that  $(z_5, +_5, \times_5)$  is a ring w.r.t.  $+_5$  and  $\times_5$ .

OR

- Prove that every field is an integral domain.
  - Show that the set of all real numbers of the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers is a ring w.r.to addition and multiplication. P.T.O.



4. a) If  $f: R \rightarrow R'$  be a homomorphism and onto then prove that  $f$  is one-one iff  $\text{Ker } f = \{0\}$ .
- b) Prove that the set  $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} / a, b \in Z \right\}$  of all  $2 \times 2$  matrices is a left ideal of the ring  $R$  over  $Z$ . Also show that  $S$  is not a right ideal.

OR

5. a) State and prove fundamental theorem of homomorphism of rings.
- b) Find all the principal ideals of the ring  $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$  w.r.to  $+$  and  $\times_8$ .

PART - C

Answer **two full** questions :

(2×10=20)

6. a) Find the directional derivative of  $\phi(x, y, z) = x^2 - y^2 + 4z^2$  at the point  $(1, 1, -8)$  in the direction of  $2\hat{i} + \hat{j} - \hat{k}$ .
- b) Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $x^2 + y^2 - z = 3$  at the point  $(2, -1, 2)$ .

OR

7. a) Prove that  $\nabla^2 r^n = n(n+1)r^{n-2}$ , where  $n$  is a non-zero constant. Also deduce that  $r^n$  is harmonic if  $n = -1$ .
- b) If the vector  $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$  is solenoidal, then find  $a$ .
8. a) If  $\phi$  is a scalar point function and  $\vec{F}$  is a vector point function. Then prove that  $\text{div}(\phi\vec{F}) = \phi(\text{div}\vec{F}) + \nabla\phi \cdot \vec{F}$ .
- b) Show that  $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$  is irrotational. Find  $\phi$  such that  $\vec{F} = \nabla\phi$ .

OR

9. a) Prove that :
- Curl  $\vec{F}$  is solenoidal.
  - Grad  $\phi$  is irrotational.
- b) Prove that  $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$  where  $r^2 = x^2 + y^2 + z^2$ .



PART - D

Answer **two** full questions.

(2×10=20)

10. a) By the separation of symbols, prove that

$$u_0 + \frac{u_1}{1!} + \frac{u_2 x^2}{2!} + \dots = e^x \left[ u_0 + \frac{x \Delta u_0}{1!} + \frac{x^2 \Delta^2 u_0}{2!} + \dots \right]$$

b) Obtain the function whose first difference is  $6x^2 + 10x + 11$ .

OR

11. a) From the following data find 'θ' at  $x = 84$  using difference table.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

b) Express  $3x^3 - 4x^2 + 3x - 11$  in factorial notation. Also express its successive differences in factorial notation.

12. a) Prepare divided difference table for the following data.

x	1	3	4	6	10
f(x)	0	18	58	190	920

b) Evaluate  $\int_0^6 \frac{1}{1+x^2} dx$ , by using Simpson's  $\frac{3}{8}$ <sup>th</sup> rule.

OR

13. a) By using Lagrange interpolation formula find  $f(10)$  from the following data.

x	5	6	9	11
f(x)	12	13	14	16

b) Evaluate  $\int_0^{0.5} e^{-x^2} dx$  by taking 6 sub intervals, by using Simpson's  $\frac{1}{3}$ <sup>rd</sup> rule.